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SELF MOTION OF A BODY IN A FLUID

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1. Many bodies (ships, living creatures) are capable of self-motion in a fluid, i.e., they move themselves by pushing fluid away from them.

The well-known (see [1], for example) equations of motion of a rigid body with respect to an inertial reference frame are

$$\frac{d\mathbf{P}_b}{dt} = \mathbf{F};\ \frac{d\mathbf{L}_b}{dt} = \mathbf{N},$$

where t is the time, $\mathbf{P}_b$ is the momentum of the body, $\mathbf{F}$ is the total external force acting on the body, $\mathbf{L}_b$ is the angular momentum of the body about the point O (the origin of the coordinate system), $\mathbf{N}$ is the total external torque acting on the body about point O. Therefore in the case when the body translates by pushing away the surrounding fluid we must have

$$\frac{d\mathbf{P}_b}{dt} = \mathbf{S}_{f\to b} \tag{1.1}$$

$$\frac{d\mathbf{L}_b}{dt} = \mathbf{T}_{f\to b} \tag{1.2}$$

where $\mathbf{S}_{f\to b}$ is the momentum transferred by the fluid to the body per unit time and $\mathbf{T}_{f\to b}$ is the angular momentum transferred by the fluid to the body per unit time about point O. Equations (1.1) and (1.2) are the basic equations describing self-motion of a body in a fluid.

In the presence of body forces, the total force acting on the body must be added to the right hand side of (1.1) and the total moment of the forces (torque) about point O must be added to the right hand side of (1.2).

Self-motion of a body in a fluid is possible because of the interaction between the boundary of the body and the fluid (but not as the result of any disturbances in the fluid which could also occur in the absence of the body). Hence the boundary of the self-moving body serves as its driver. In self-motion the operation of the driver is such that conditions exist on the boundary of the body for which the equations of self-motion are satisfied.

2. An approximate solution of the problem was found in [2, 3] for steady flow of a viscous incompressible fluid past a self-moving body (a circular cylinder and a sphere). In the

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present paper we consider the unsteady flow of a viscous incompressible fluid past a self-moving body.

The self-moving body is assumed to be a homogeneous sphere with a moving boundary. The velocity $\mathbf{U}$ of the boundary relative to the sphere varies periodically in time with period T. The motion of the fluid and sphere are considered with respect to a nonrotating system of rectangular coordinates X_1, X_2, X_3 with the origin of the coordinate system at the center of the sphere. The sphere rotates about the X_3 axis. The motion of the fluid is symmetric about the X_3 axis, is steady (it does not depend on the initial conditions) and varies in time periodically with period T.

Let $\tau = t/T$, A be the radius of the sphere, $x_1 = X_1/A$, $x_2 = X_2/A$, $x_3 = X_3/A$; $\mathbf{r} = (x_1, x_2, x_3)$; r, θ, φ are spherical coordinates related to x_1, x_2, x_3 by the relations $x_1 = r\sin\theta\cos\varphi$, $x_2 = r\sin\theta\sin\varphi$, $x_3 = r\cos\theta$; ρ_{sph} is the density of the sphere, ρ_f is the density of the fluid, $\rho = \rho_{sph}/\rho_f$, $\mathbf{k} = (0, 0, 1)$, $W\mathbf{k}$ is the velocity of the center of the sphere with respect to the fluid at infinity, $w = TW/A$, $\Omega\mathbf{k}$ is the angular velocity of the sphere, $\omega = T\Omega$, $\mathbf{V}$ is the velocity of the fluid, $\mathbf{v} = T\mathbf{V}/A$, v_r, v_θ, v_φ are the r, θ, φ components of the vector $\mathbf{v}$, P is the fluid pressure, P_∞ is the fluid pressure at infinity, $p = T^2(P - P_\infty)/(\rho_f A^2)$, U_θ and U_φ are the θ and φ components of the vector $\mathbf{U}$, U is the magnitude $|\mathbf{U}|$; $u_\theta = U_\theta/U$ $(u_\theta = u_\theta(\theta, \tau))$; $u_\varphi = U_\varphi/U$ $(u_\varphi = u_\varphi(\theta, \tau))$; $\varepsilon = UT/A$; ν is the kinematic viscosity of the fluid, $\mathrm{Re} = A^2/(\nu T)$ is the Reynolds number, $\mathbf{P}$ is the stress tensor in the fluid, $\mathbf{p} = T^2\mathbf{P}/(\rho_f A^2)$; $\mathbf{n}$ is the unit outward normal to the surface, s is the sphere $r = 1$, $\nabla = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$; $\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2$.

The equations of self-motion of the sphere, the Navier–Stokes equations, the equation of continuity, and the boundary conditions at the surface of the sphere and at infinity can be written as

$$\iint_s \mathbf{p}\cdot\mathbf{n}\,ds - \frac{4\pi}{3}\rho\frac{dw}{d\tau}\mathbf{k} = 0; \qquad (2.1)$$

$$\iint_s \mathbf{r}\times(\mathbf{p}\cdot\mathbf{n})\,ds - \frac{8\pi}{15}\rho\frac{d\omega}{d\tau}\mathbf{k} = 0; \qquad (2.2)$$

$$\frac{\partial\mathbf{v}}{\partial\tau} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla p - \frac{1}{\mathrm{Re}}\Delta\mathbf{v} + \frac{dw}{d\tau}\mathbf{k} = 0; \qquad (2.3)$$

$$\nabla\cdot\mathbf{v} = 0; \qquad (2.4)$$

$$v_r = 0,\ v_\theta = \varepsilon u_\theta,\ v_\varphi = \varepsilon u_\varphi + \omega\sin\theta \ \text{ for } r = 1; \qquad (2.5)$$

$$\mathbf{v} \sim -w\mathbf{k},\ p \sim 0 \ \text{ for } r\to\infty. \qquad (2.6)$$

It is required to find w, ω, $\mathbf{v}$, p.

3. We will consider the problem (2.1)-(2.6) for values of ε which are small in comparison with unity. We assume that when $\varepsilon \to 0$

$$w(\tau, \varepsilon, \mathrm{Re}, \rho) \sim \sum_{n=1}^{6} \varepsilon^n w_{(n)}(\tau, \mathrm{Re}, \rho); \qquad (3.1)$$

$$\omega(\tau, \varepsilon, \mathrm{Re}, \rho) \sim \varepsilon\omega_{(1)}(\tau, \mathrm{Re}, \rho); \qquad (3.2)$$

$$\mathbf{v}(\mathbf{r}, \tau, \varepsilon, \mathrm{Re}, \rho) \sim \varepsilon\mathbf{v}_{(1)}(\mathbf{r}, \tau, \mathrm{Re}, \rho); \qquad (3.3)$$

$$p(\mathbf{r}, \tau, \varepsilon, \mathrm{Re}, \rho) \sim \varepsilon p_{(1)}(\mathbf{r}, \tau, \mathrm{Re}, \rho). \qquad (3.4)$$

The asymptotic expansions obtained in the limit $\varepsilon \to 0$ and constant $\mathbf{r}$, τ, Re, ρ will be called inner expansions. Using (2.1)-(2.6) and (3.1)-(3.4) we find

$$\frac{1}{\mathrm{Re}}\int_0^\pi \left[\left(-\mathrm{Re}\,p_{(1)} + 2\frac{\partial v_{(1)r}}{\partial r}\right)\cos\theta - \left(\frac{\partial v_{(1)\theta}}{\partial r} - u_\theta\right)\sin\theta\right]\bigg|_{r=1}\sin\theta\,d\theta - \frac{2}{3}\rho\frac{dw_{(1)}}{d\tau} = 0; \qquad (3.5)$$

$$\frac{1}{\mathrm{Re}}\int_0^\pi \left(\frac{\partial v_{(1)\varphi}}{\partial r} - u_\varphi - \omega_{(1)}\sin\theta\right)\bigg|_{r=1}\sin^2\theta\,d\theta - \frac{4}{15}\rho\frac{d\omega_{(1)}}{d\tau} = 0; \qquad (3.6)$$

$$\frac{\partial\mathbf{v}_{(1)}}{\partial\tau} + \nabla p_{(1)} - \frac{1}{\mathrm{Re}}\Delta\mathbf{v}_{(1)} + \frac{dw_{(1)}}{d\tau}\mathbf{k} = 0; \qquad (3.7)$$

$$\nabla \cdot \mathbf{v}_{(1)} = 0; \tag{3.8}$$

$$v_{(1)r} = 0,\ v_{(1)\theta} = u_\theta,\ v_{(1)\varphi} = u_\varphi + \omega_{(1)} \sin\theta \ \text{ for } r = 1; \tag{3.9}$$

$$\mathbf{v}_{(1)} \sim -w_{(1)}\mathbf{k},\ p_{(1)} \sim 0 \qquad \text{for } r \to \infty, \tag{3.10}$$

where $v_{(1)r}$, $v_{(1)\theta}$, $v_{(1)\varphi}$ are the r, θ, φ components of the vector $\mathbf{v}_{(1)}$.

We write u_θ and u_φ in the form of the series

$$u_\theta = \sum_{l=1}^{\infty}\left(\eta_{l0} + \text{Real}\sum_{m=1}^{\infty}\eta_{lm}e^{2m\pi i\tau}\right)P_l^{(1)}(\cos\theta);$$

$$u_\varphi = \sum_{l=1}^{\infty}\left(\xi_{l0} + \text{Real}\sum_{m=1}^{\infty}\xi_{lm}e^{2m\pi i\tau}\right)P_l^{(1)}(\cos\theta),$$

where $\eta_{\ell 0}$, $\eta_{\ell m}$, $\xi_{\ell 0}$, $\xi_{\ell m}$ are constants ($\eta_{10} < 0$, $\eta_{20} \neq 0$, $\xi_{20} \neq 0$); $P_\ell^{(1)}$ are the associated Legendre functions.

The problem (3.5)-(3.10) has the solution

$$w_{(1)} = w_{(1)0} + \text{Real}\sum_{m=1}^{\infty} w_{(1)m}e^{2m\pi i\tau}; \tag{3.11}$$

$$\omega_{(1)} = \omega_{(1)0} + \text{Real}\sum_{m=1}^{\infty}\omega_{(1)m}e^{2m\pi i\tau}; \tag{3.12}$$

$$v_{(1)r} = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta},\ v_{(1)\theta} = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}; \tag{3.13}$$

$$v_{(1)\varphi} = \sum_{l=1}^{\infty}\left[(\xi_{l0} + \omega_{(1)0}\delta_{l1})r^{-l-1} + \text{Real}\sum_{m=1}^{\infty}\frac{\xi_{lm} + \omega_{(1)m}\delta_{l1}}{K_{l+1/2}(q_m)}r^{-1/2}K_{l+1/2}(q_m r)\,e^{2m\pi i\tau}\right]P_l^{(1)}(\cos\theta); \tag{3.14}$$

$$p_{(1)} = \sum_{l=1}^{\infty}\left\{-\frac{\partial^2}{\partial\tau\partial r}\psi_l + \frac{1}{\text{Re}}\left[\frac{\partial^3}{\partial r^3} - \frac{n(n+1)}{r^2}\frac{\partial}{\partial r} + \frac{2n(n+1)}{r^3}\right]\psi_l - \frac{dw_{(1)}}{d\tau}r\delta_{l1}\right\}P_l(\cos\theta) \tag{3.15}$$

$$\left(w_{(1)0} = \frac{2}{3}\eta_{10};\ w_{(1)m} = 6\frac{q_m + 1}{(2\rho+1)q_m^2 + 9q_m + 9}\eta_{1m};\right.$$

$$\omega_{(1)0} = -\xi_{10};\ \omega_{(1)m} = -5\frac{q_m^2 + 3q_m + 3}{\rho q_m^3 + (\rho+5)q_m^2 + 15q_m + 15}\xi_{1m};$$

$$\psi = \sum_{l=1}^{\infty}\psi_l P_l^{(1)}(\cos\theta)\sin\theta;\ \psi_l = -\frac{1}{2}w_{(1)}r^2\delta_{l1} + \frac{1}{4}\left[(2\eta_{l0} - w_{(1)0}\delta_{l1})r^{-l} -\right.$$

$$\left.-(2\eta_{l0} - 3w_{(1)0}\delta_{l1})r^{-l+2}\right] - \frac{1}{2}\text{Real}\sum_{m=1}^{\infty}\left\{\left[2q_m\frac{K_{l+1/2}(q_m)}{K_{l-1/2}(q_m)}\eta_{lm} -\right.\right.$$

$$\left.\left.-(q_m^2 + 3q_m + 3)w_{(1)m}\delta_{l1}\right]r^{-l} - q_m\frac{2\eta_{lm} - 3w_{(1)m}\delta_{l1}}{K_{l-1/2}(q_m)}r^{1/2}K_{l+1/2}(q_m r)\right\}\frac{e^{2m\pi i\tau}}{q_m^2};$$

$q_m = (1 + i)\sqrt{m\pi \text{Re}}$; δ is the Kronecker delta, P_ℓ are the Legendre polynomials, $K_{\ell\pm1/2}$ are the MacDonald functions).

The relations

$$w = \varepsilon w_{(1)},\ \omega = \varepsilon\omega_{(1)},\ \mathbf{v} = \varepsilon\mathbf{v}_{(1)},\ p = \varepsilon p_{(1)} \tag{3.16}$$

and (3.11)-(3.15) determine the approximate solution of the problem (2.1)-(2.6).

4. The solution (3.11)-(3.16) satisfies the boundary condition (2.6) at infinity exactly. However, it incorrectly characterizes the disturbances of the motion of the fluid for $r \gtrsim 1/\varepsilon$ [using (3.13), (3.14), and (3.16) it is not difficult to show that when $r \gtrsim 1/\varepsilon$ the quantity $(\mathbf{v}\cdot\nabla)\mathbf{v}$ cannot be neglected in (2.3)].

We determine approximate expressions for the velocity disturbance $\mathbf{v} + w\mathbf{k}$ and pressure disturbance p in the entire region occupied by the fluid. Together with the inner expansions

(3.3) and (3.4) we will also consider the outer expansions of v and p. The inner and outer expansions of v and p must be consistent with one another in correspondence with the matching principle for asymptotic expansions [4].

We write (2.3), (2.4), and the condition (2.6) in the form

$$\frac{\partial \mathbf{v}}{\partial \tau}+\varepsilon(\mathbf{v}\cdot\hat{\nabla})\mathbf{v}+\varepsilon\hat{\nabla}p-\frac{\varepsilon^2}{\mathrm{Re}}\hat{\Delta}\mathbf{v}+\frac{dw}{d\tau}\mathbf{k}=0; \tag{4.1}$$

$$\hat{\nabla}\cdot\mathbf{v}=0; \tag{4.2}$$

$$\mathbf{v}\sim -w\mathbf{k},\ p\sim 0 \text{ for } \hat{r}\to\infty, \tag{4.3}$$

where $\hat{\nabla} = (\partial/\partial\hat{x}_1,\ \partial/\partial\hat{x}_2,\ \partial/\partial\hat{x}_3)$; $\hat{\Delta} = \partial^2/\partial\hat{x}_1{}^2 + \partial^2/\partial\hat{x}_2{}^2 + \partial^2/\partial\hat{x}_3{}^2$ ($\hat{x}_1 = \varepsilon x_1$, $\hat{x}_2 = \varepsilon x_2$, $\hat{x}_3 = \varepsilon x_3$); $\hat{r} = \varepsilon r$. We assume that in the limit $\varepsilon \to 0$

$$\mathbf{v}\left(\frac{\hat{\mathbf{r}}}{\varepsilon},\tau,\varepsilon,\mathrm{Re},\rho\right)\sim-\varepsilon w_{(1)}\mathbf{k}+\sum_{n=2}^{6}\varepsilon^n\mathbf{v}^{(n)}(\hat{\mathbf{r}},\tau,\mathrm{Re},\rho); \tag{4.4}$$

$$p\left(\frac{\hat{\mathbf{r}}}{\varepsilon},\tau,\varepsilon,\mathrm{Re},\rho\right)\sim\sum_{n=2}^{6}\varepsilon^{n-1}p^{(n)}(\hat{\mathbf{r}},\tau,\mathrm{Re},\rho), \tag{4.5}$$

where $\hat{\mathbf{r}}=\varepsilon\mathbf{r}$. Asymptotic expansions obtained in the limit $\varepsilon \to 0$ and constant $\hat{\mathbf{r}}$, τ, Re, ρ will be called outer expansions. Using (4.1)-(4.5) we find

$$\frac{\partial\mathbf{v}^{(K)}}{\partial\tau}+\hat{\nabla}p^{(K)}+\frac{dw_{(K)}}{d\tau}\mathbf{k}-\left[w_{(1)}(\mathbf{k}\cdot\hat{\nabla})\mathbf{v}^{(2)}+\frac{1}{\mathrm{Re}}\hat{\Delta}\mathbf{v}^{(2)}\right]\delta_{K4}-\left[w_{(1)}(\mathbf{k}\cdot\hat{\nabla})\mathbf{v}^{(3)}-(\mathbf{v}^{(2)}\cdot\hat{\nabla})\mathbf{v}^{(2)}+\frac{1}{\mathrm{Re}}\hat{\Delta}\mathbf{v}^{(3)}\right]\delta_{K5}-$$
$$-\left[w_{(1)}(\mathbf{k}\cdot\hat{\nabla})\mathbf{v}^{(4)}-(\mathbf{v}^{(2)}\cdot\hat{\nabla})\mathbf{v}^{(3)}-(\mathbf{v}^{(3)}\cdot\hat{\nabla})\mathbf{v}^{(2)}+\frac{1}{\mathrm{Re}}\hat{\Delta}\mathbf{v}^{(4)}\right]\delta_{K6}=0; \tag{4.6}$$

$$\hat{\nabla}\cdot\mathbf{v}^{(K)}=0; \tag{4.7}$$

$$\mathbf{v}^{(K)}\sim-w_{(K)}\mathbf{k},\ p^{(K)}\sim 0 \text{ for } \hat{r}\to\infty \tag{4.8}$$

$(K = 2, 3, 4, 5, 6)$.

The matching conditions for the inner and outer expansions of **v** and p are

$$I_\varepsilon E_{\varepsilon L}\mathbf{v}=E_{\varepsilon L}I_\varepsilon\mathbf{v} \qquad (L=1,2,3,4,5,6); \tag{4.9}$$

$$I_\varepsilon E_{\varepsilon M}p=E_{\varepsilon M}I_\varepsilon p \qquad (M=0,1,2,3,4,5), \tag{4.10}$$

where I and E are operators corresponding to the inner and outer expansions, respectively (see [2, 5]).

Using (3.3), (3.4), (3.13)-(3.15), (4.4), (4.5), we find that the conditions (4.9) and (4.10) are satisfied when L = 1, M = 0.

We write $w_{(k)}$, $\mathbf{v}^{(k)}$, $p^{(k)}$ (k = 2, 3, 4, 5) in the form of the series

$$w_{(K)}=w_{(K)0}+\mathrm{Real}\sum_{m=1}^{\infty}w_{(K)m}e^{2m\pi i\tau};$$

$$\mathbf{v}^{(K)}=\mathbf{v}_0^{(K)}+\mathrm{Real}\sum_{m=1}^{\infty}\mathbf{v}_m^{(K)}e^{2m\pi i\tau};$$

$$p^{(K)}=p_0^{(K)}+\mathrm{Real}\sum_{m=1}^{\infty}p_m^{(K)}e^{2m\pi i\tau},$$

where $w_{(k)0}$, $w_{(k)m}$ are constants and $\mathbf{v}_0^{(K)}$, $\mathbf{v}_m^{(K)}$, $p_0^{(K)}$, $p_m^{(K)}$ are functions of $\hat{\mathbf{r}}$.

It follows from (4.6)-(4.10) with K = 2, L = 2, M = 1 and (3.3), (3.4), (3.13)-(3.15), (4.4), (4.5) that

$$\mathbf{v}_m^{(2)}=-w_{(2)m}\mathbf{k} \quad (m=1,2,\ldots); \tag{4.11}$$

$$p^{(2)}=0; \tag{4.12}$$

$$\widehat{\nabla}\cdot\mathbf{v}_0^{(2)}=0; \tag{4.13}$$

$$\mathbf{v}_0^{(2)}\sim -w_{(2)0}\mathbf{k} \quad \text{for} \quad \widehat{r}\to\infty; \tag{4.14}$$

$$I_\varepsilon\left(\varepsilon^2 v_{0r}^{(2)}\right)=0,\quad I_\varepsilon\left(\varepsilon^2 v_{0\theta}^{(2)}\right)=0,\quad I_\varepsilon\left(\varepsilon^2 v_{0\varphi}^{(2)}\right)=0, \tag{4.15}$$

where $v_{0r}{}^{(2)}$, $v_{0\theta}{}^{(2)}$, $v_{0\varphi}{}^{(2)}$ are the r, θ, φ components of the vector $\mathbf{v}_0^{(2)}$. Using (4.6) with K = 4 and (3.11) and (4.11), we obtain

$$\mathrm{Re}\,\widehat{\nabla}p_0^{(4)}+2\lambda\left(\mathbf{k}\cdot\widehat{\nabla}\right)\mathbf{v}_0^{(2)}-\widehat{\Delta}\mathbf{v}_0^{(2)}=0, \tag{4.16}$$

where $\lambda = -\mathrm{Re}\,\eta_{10}/3$. The problem (4.13)-(4.16) has the solution

$$\mathbf{v}_0^{(2)}=-w_{(2)0}\mathbf{k}; \tag{4.17}$$

$$p_0^{(4)}=c \tag{4.18}$$

(c is a constant).

It follows from (4.6)-(4.10) with K = 3, L = 3, M = 2 and (3.3), (3.4), (3.13)-(3.15), (4.4), (4.5) that

$$\mathbf{v}_m^{(3)}=-w_{(3)m}\mathbf{k}\ (m=1,2,\ldots); \tag{4.19}$$

$$p^{(3)}=0; \tag{4.20}$$

$$\widehat{\nabla}\cdot\mathbf{v}_0^{(3)}=0; \tag{4.21}$$

$$\mathbf{v}_0^{(3)}\sim -w_{(3)0}\mathbf{k}\ \ \text{for}\ \ \widehat{r}\to\infty; \tag{4.22}$$

$$\begin{gathered}I_\varepsilon\left(\varepsilon^3 v_{0r}^{(3)}\right)=-3\varepsilon\eta_{20}r^{-2}P_2(\cos\theta),\\ I_\varepsilon\left(\varepsilon^3 v_{0\theta}^{(3)}\right)=0,\quad I_\varepsilon\left(\varepsilon^3 v_{0\varphi}^{(3)}\right)=0,\end{gathered} \tag{4.23}$$

where $v_{0r}{}^{(3)}$, $v_{0\theta}{}^{(3)}$, $v_{0\varphi}{}^{(3)}$ are the r, θ, φ components of the vector $\mathbf{v}_0^{(3)}$. Using (4.6) with K = 5 and (3.11), (4.11), (4.17), (4.19), we obtain

$$\mathrm{Re}\,\widehat{\nabla}p_0^{(5)}+2\lambda\left(\mathbf{k}\cdot\widehat{\nabla}\right)\mathbf{v}_0^{(3)}-\widehat{\Delta}\mathbf{v}_0^{(3)}=0. \tag{4.24}$$

The problem (4.21)-(4.24) has the solution

$$\begin{gathered}v_{0r}^{(3)}=\frac{\partial\Phi}{\partial\widehat{r}}+\frac{1}{2\lambda}\frac{\partial\chi}{\partial\widehat{r}}-\chi\cos\theta,\\ v_{0\theta}^{(3)}=\frac{1}{\widehat{r}}\frac{\partial\Phi}{\partial\theta}+\frac{1}{2\lambda\widehat{r}}\frac{\partial\chi}{\partial\theta}+\chi\sin\theta;\end{gathered} \tag{4.25}$$

$$v_{0\varphi}^{(3)}=0; \tag{4.26}$$

$$p_0^{(5)}=\frac{2\lambda}{\mathrm{Re}}\left(-\frac{\partial\Phi}{\partial\widehat{r}}\cos\theta+\frac{1}{\widehat{r}}\sin\theta\right)+c' \tag{4.27}$$

$$\left(\Phi=-\left(w_{(3)0}\widehat{r}+\frac{3\eta_{20}}{2\lambda\widehat{r}^2}\right)\cos\theta+\frac{\alpha}{\widehat{r}};\quad \chi=-\frac{\lambda}{\widehat{r}}e^{\lambda\widehat{r}(\cos\theta-1)}\left[3\eta_{20}+2\alpha-\right.\right.$$
$$\left.\left.-3\eta_{20}\left(1+\frac{1}{\lambda\widehat{r}}\right)\cos\theta\right];\quad c',\alpha-\text{constant}\right).$$

It follows from (4.6), (4.8), (4.9) with K = 4, L = 4 and (3.3), (3.14), (4.4), (4.11), (4.17) that

$$\frac{\partial v_\varphi^{(4)}}{\partial\tau}=0; \tag{4.28}$$

$$v_\varphi^{(4)}\sim 0\ \ \text{for}\ \ \widehat{r}\to\infty; \tag{4.29}$$

$$I_\varepsilon\left(\varepsilon^4 v_\varphi^{(4)}\right)=\varepsilon\xi_{20}r^{-3}P_2^{(1)}(\cos\theta), \tag{4.30}$$

where $v_\varphi{}^{(4)}$ is the φ component of the vector $\mathbf{v}^{(4)}$. Using (4.6) with K = 6 and (3.11), (4.11), (4.17), (4.19), (4.26), (4.28), we obtain

$$2\lambda\mathbf{k}\cdot\hat{\nabla}v_\varphi^{(4)} - \hat{\Delta}v_\varphi^{(4)} + \frac{v_\varphi^{(4)}}{\hat{r}^2\sin^2\theta} = 0. \quad (4.31)$$

The problem (4.29)-(4.31) has the solution

$$v_\varphi^{(4)} = \frac{\lambda^2}{\hat{r}} e^{\lambda\hat{r}(\cos\theta-1)}\left[\beta\left(1+\frac{1}{\lambda\hat{r}}\right) + \xi_{20}\left(1+\frac{3}{\lambda\hat{r}}+\frac{3}{\lambda^2\hat{r}^2}\right)\cos\theta\right]\sin\theta \quad (4.32)$$

(β is a constant).

The momentum of the sphere and fluid inside a closed surface σ containing the sphere, and the angular momentum of the sphere and fluid about the origin of the coordinate system $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$ vary periodically with τ with period 1. Hence, we must have the relations

$$\int_\tau^{\tau+1}\iint_\sigma [\mathbf{p}\cdot\mathbf{n} - \mathbf{v}(\mathbf{v}\cdot\mathbf{n})]\, d\sigma\, d\tau = 0; \quad (4.33)$$

$$\int_\tau^{\tau+1}\iint_\sigma \{\hat{\mathbf{r}}\times[\mathbf{p}\cdot\mathbf{n} - \mathbf{v}(\mathbf{v}\cdot\mathbf{n})]\}\, d\sigma\, d\tau = 0. \quad (4.34)$$

Using (3.11), (4.4), (4.5), (4.11), (4.12), (4.17)-(4.20), (4.25)-(4.27), (4.32)-(4.34), we obtain

$$\alpha = 0,\ \beta = -\xi_{20}. \quad (4.35)$$

Applying the additive method for the inner and outer expansions [4], we find that in the entire region occupied by the fluid the r, θ, φ components of the vector $\mathbf{v} + w\mathbf{k}$ and p are given approximately by the relations

$$v_r + w\cos\theta = \varepsilon(v_{(1)r} + w_{(1)}\cos\theta) + \varepsilon^3\left(v_{0r}^{(3)} + w_{(3)0}\cos\theta\right) + \frac{3}{4}\varepsilon\eta_{20}r^{-2}(1+3\cos 2\theta); \quad (4.36)$$

$$v_\theta - w\sin\theta = \varepsilon(v_{(1)\theta} - w_{(1)}\sin\theta) + \varepsilon^3\left(v_{0\theta}^{(3)} - w_{(3)0}\sin\theta\right); \quad (4.37)$$

$$v_\varphi = \varepsilon v_{(1)\varphi} + \varepsilon^4 v_\varphi^{(4)} - \frac{3}{2}\varepsilon\xi_{20}r^{-3}\sin 2\theta; \quad (4.38)$$

$$p = \varepsilon p_{(1)}$$

and by (3.13)-(3.15), (4.25), (4.32), (4.35).

5. We consider the asymptotic behavior of the velocity disturbance of the fluid at large distances from the sphere (small ε). Using (3.13), (3.14), (4.25), (4.32), (4.35)-(4.38), we obtain

$$v_r + w\cos\theta \sim \frac{3\varepsilon\eta_{20}}{x_3^2}\left(\frac{\varepsilon\lambda}{2}\frac{x_1^2+x_2^2}{x_3} - 1\right)e^{-\frac{\varepsilon\lambda}{2}\frac{x_1^2+x_2^2}{x_3}},$$

$$v_\theta - w\sin\theta \sim -\frac{3\varepsilon^2\lambda\eta_{20}}{4x_3^{5/2}}\left(\frac{x_1^2+x_2^2}{x_3}\right)^{3/2}e^{-\frac{\varepsilon\lambda}{2}\frac{x_1^2+x_2^2}{x_3}}, \quad (5.1)$$

$$v_\varphi \sim \frac{2\varepsilon^2\lambda\xi_{20}}{x_3^{5/2}}\left(\frac{x_1^2+x_2^2}{x_3}\right)^{1/2}\left(1 - \frac{\varepsilon\lambda}{4}\frac{x_1^2+x_2^2}{x_3}\right)e^{-\frac{\varepsilon\lambda}{2}\frac{x_1^2+x_2^2}{x_3}}$$

in the limit $x_3 \to +\infty$ and constant $(x_1^2 + x_2^2)/x_3$, τ, ε, Re, ρ. According to (5.1), the velocity disturbance becomes steady at large distances from the sphere and falls off as X_3^{-2}.

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